3. To Commemorate the Centenary of Rutherford’s Atomic Nucleus:  
the Scattering of an Ion by a Neutral Atom

An ion of mass \( m \), charge \( Q \), is moving with an initial non-relativistic speed \( v_0 \) from a great distance towards the vicinity of a neutral atom of mass \( M >> m \) and of electrical polarisability \( \alpha \). The impact parameter is \( b \) as shown in Figure 1.

The atom is instantaneously polarised by the electric field \( \vec{E} \) of the in-coming (approaching) ion. The resulting electric dipole moment of the atom is \( \vec{p} = \alpha \vec{E} \). Ignore any radiative losses in this problem.

3.1 Calculate the electric field intensity \( \vec{E}_p \) at a distance \( r \) from an ideal electric dipole \( \vec{p} \) at the origin O along the direction of \( \vec{p} \) in Figure 2.  

\[
p = 2aq, \quad r \gg a
\]

FIGURE 1

FIGURE 2
3.2 Find the expression for the force $\vec{f}$ acting on the ion due to the polarised atom. Show that this force is attractive regardless of the sign of the charge of the ion. [3.0 points]

3.3 What is the electric potential energy of the ion-atom interaction in terms of $\alpha, Q$ and $r$? [0.9 points]

3.4 Find the expression for $r_{\text{min}}$, the distance of the closest approach, as shown in Figure 1. [2.4 points]

3.5 If the impact parameter $b$ is less than a critical value $b_0$, the ion will descend along a spiral to the atom. In such a case, the ion will be neutralized, and the atom is, in turn, charged. This process is known as the “charge exchange” interaction. What is the cross sectional area $A = \pi b_0^2$ of this “charge exchange” collision of the atom as seen by the ion? [2.5 points]