2. An Electrified Soap Bubble

A spherical soap bubble with internal air density ρ_i, temperature T_i and radius R_0 is surrounded by air with density ρ_a, atmospheric pressure P_a and temperature T_a. The soap film has surface tension γ, density ρ_s and thickness t. The mass and the surface tension of the soap do not change with the temperature. Assume that $R_0 \gg t$.

The increase in energy, dE, that is needed to increase the surface area of a soap-air interface by dA, is given by $dE = \gamma dA$ where γ is the surface tension of the film.

2.1 Find the ratio $\frac{\rho_i T_i}{\rho_a T_a}$ in terms of γ, P_a and R_0. **[1.7 point]**

2.2 Find the numerical value of $\frac{\rho_i T_i}{\rho_a T_a} - 1$ using $\gamma = 0.0250\,\mathrm{Nm}^{-1}$, $R_0 = 1.00\ \mathrm{cm}$, and $P_a = 1.013 \times 10^5\,\mathrm{Nm}^{-2}$. **[0.4 point]**

2.3 The bubble is initially formed with warmer air inside. Find the minimum numerical value of T_i such that the bubble can float in still air. Use $T_a = 300\ \mathrm{K}$, $\rho_s = 1000\ \ \mathrm{kgm}^{-3}$, $\rho_a = 1.30\ \ \mathrm{kgm}^{-3}$, $t = 100\ \mathrm{nm}$ and $g = 9.80\ \ \mathrm{ms}^{-2}$. **[2.0 points]**

After the bubble is formed for a while, it will be in thermal equilibrium with the surrounding. This bubble in still air will naturally fall towards the ground.

2.4 Find the minimum velocity u of an updraught (air flowing upwards) that will keep the bubble from falling at thermal equilibrium. Give your answer in terms of ρ_s, R_0, g, t and the air's coefficient of viscosity η. You may assume that the velocity is small such that Stokes's law applies, and ignore the change in the radius when the temperature lowers to the equilibrium. The drag force from Stokes' Law is $F = 6\pi\eta R_0 u$. **[1.6points]**

2.5 Calculate the numerical value for u using $\eta = 1.8 \times 10^{-5}\ \mathrm{kgm}^{-1}\,\mathrm{s}^{-1}$. **[0.4 point]**

The above calculations suggest that the terms involving the surface tension γ add very little to the accuracy of the result. In all of the questions below, you can neglect the surface tension terms.

2.6 If this spherical bubble is now electrified uniformly with a total charge q, find an equation describing the new radius R_1 in terms of R_0, P_a, q and the permittivity of free space ε_0.

[2.0points]

2.7 Assume that the total charge is not too large (i.e. $\frac{q^2}{\varepsilon_0 R_0^4} << P_a$) and the bubble only experiences a small increase in its radius, find ΔR where $R_1 = R_0 + \Delta R$.

Given that $(1+x)^n \approx 1 + nx$ where $x \ll 1$. **[0.7 point]**

2.8 What must be the magnitude of this charge q in terms of $t, \rho_a, \rho_s, \varepsilon_0, R_0, P_a$ in order that the bubble will float motionlessly in still air? Calculate also the numerical value of q. The permittivity of free space $\varepsilon_0 = 8.85 \times 10^{-12}$ farad/m. **[1.2 point]**