

## 2. Mechanical Blackbox: a cylinder with a ball inside

A small massive particle (ball) of mass  $m$  is fixed at distance  $z$  below the top of a long hollow cylinder of mass  $M$ . A series of holes are drilled perpendicularly to the central axis of the cylinder. These holes are for pivoting so that the cylinder will hang in a vertical plane.

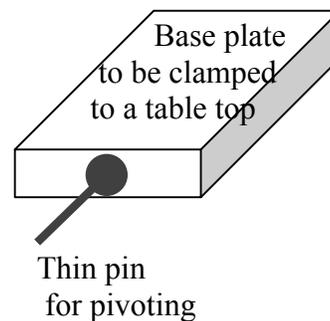
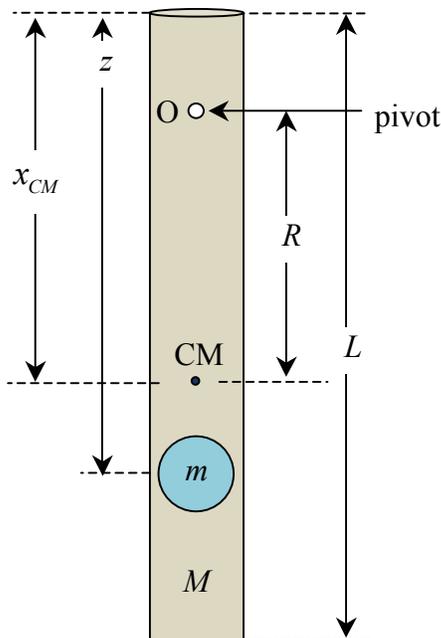
Students are required to perform necessary nondestructive measurements to determine the numerical values of the following with their error estimates:

- i. position of centre of mass of cylinder with ball inside.

Also provide a schematic drawing of the experimental set-up for measuring the centre of mass. [1.0 points]

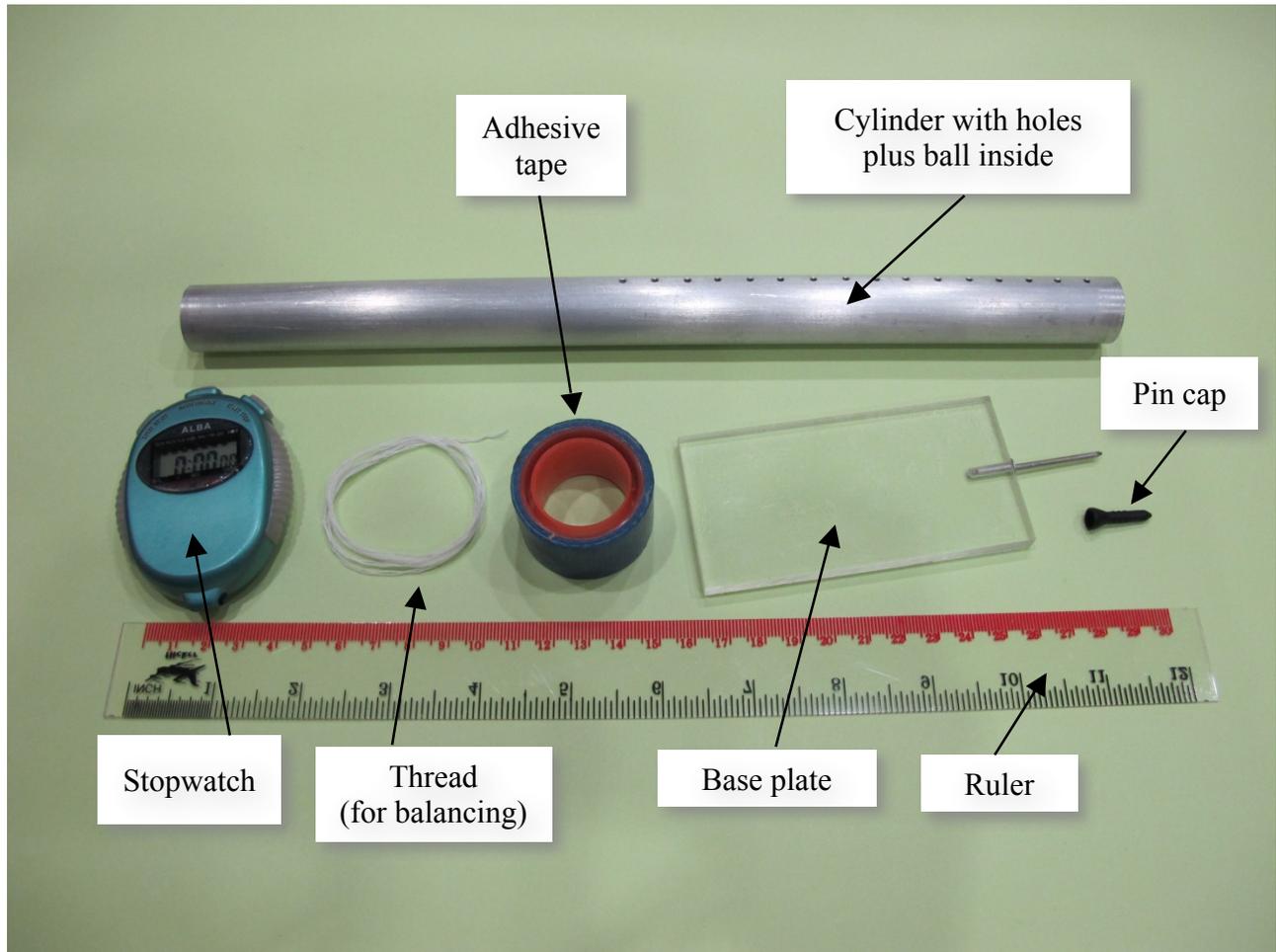
- ii. distance  $z$  [3.5 points]
- iii. ratio  $M/m$ . [3.5 points]
- iv. the acceleration due to gravity,  $g$ . [2.0 points]

**Equipment:** a cylinder with holes plus a ball inside, a base plate with a thin pin, a pin cap, a ruler, a stopwatch, thread, a pencil and adhesive tape.



$x_{CM}$  is the distance from the top of the cylinder to the centre of mass.

$R$  is the distance from the pivoting point to the centre of mass.



**Caution:** The thin pin is sharp. When it is not in use, it should be protected with a pin cap for safety.

**Useful information:**

1. For such a physical pendulum,  $\left\{ (M + m)R^2 + I_{CM} \right\} \frac{d^2\theta}{dt^2} \approx -g(M + m)R\theta$ , where  $I_{CM}$  is the moment of inertia of the cylinder with a ball about the centre of mass and  $\theta$  is the angular displacement.
2. For a long hollow cylinder of length  $L$  and mass  $M$ , the moment of inertia about the centre of mass with the rotational axis perpendicular to the cylinder can be approximated by  $\frac{1}{3}M\left(\frac{L}{2}\right)^2$ .
3. The parallel axis theorem:  $I = I_{\text{centre of mass}} + \mathfrak{M}x^2$ , where  $x$  is the distance from the rotation point to the centre of mass, and  $\mathfrak{M}$  is the total mass of the object.
4. The ball can be treated as a point mass and it is located on the central axis of the cylinder.
5. Assume that the cylinder is uniform and the mass of the end-caps is negligible.